(54) Title: PREFILTER DESIGN BY SPECTRAL FACTORIZATION

(57) Abstract: A baseband signal processing method for calculating a minimum phase version of a channel estimate using spectral factorization is disclosed. The method provides for a low complexity implementation of prefilters. The prefilter does not color the signal and operates as an allpass filter. Additionally, methods are described for constructing prefilters for equalization in both the forward and backward direction.

PREFILTER DESIGN BY SPECTRAL FACTORIZATION

BACKGROUND

The present invention relates to communication systems and in particular to baseband signal processing in the receiving parts of base stations and mobile

5   stations. The present invention provides a filter design method using spectral factorization that has a low complexity in implementation of the filter.

The cellular telephone industry has made phenomenal strides in commercial operations in the United States as well as the rest of the world. Growth in major metropolitan areas has far exceeded expectations and is rapidly outstripping system

10   capacity. If this trend continues, the effects of this industry's growth will soon reach even the smallest markets. Innovative solutions are required to meet these increasing capacity needs as well as maintain high quality service and avoid rising prices.

In mobile communication, the transmitted signal is often subjected to a time

15   smearing effect created by the time dispersive nature of the channel, i.e., the air interface between a base station and a mobile station. The channel is estimated in the receiver part of a communication system, and used by the detector to aid in attempting to correctly deduce the information symbols that were transmitted thereto. A commonly used technique for deducing such received information

20   symbols is Maximum Likelihood Sequence Estimation (MLSE) which, implemented using the Viterbi algorithm, is optimal for situations involving Additive White Gaussian Noise (AWGN).

Although the MLSE can be implemented through a computationally efficient Viterbi scheme, the MLSE might nonetheless be very computationally complex if

25   large symbol alphabets are employed or if the number of required taps in the channel estimate is large. In order to reduce the complexity of the Viterbi part of the equalizer, decision feedback can be used. In a Decision Feedback Sequence

Estimator (DFSE) equalizer, the estimated channel impulse response is split into two parts. The part which contains the first filter taps is referred to as the MLSE part and the remainder is referred to as the Decision Feedback Estimator (DFE) part.

5   By implementing this hybrid MLSE/DFE structure, the computational complexity of the detector is reduced, but with some possible loss in performance compared to a pure MLSE detector implementation. If the energy of the channel is concentrated in the first channel taps, i.e. the MLSE part, then the performance loss is small. This can be accomplished by placing a prefilter in the receive signal

10   processing chain. The purpose of the prefilter is to change the characteristic of the channel's impulse response such that the first channel taps are as large as possible. Furthermore, the prefilter should not change the magnitude response of the channel, or color the noise.

Basically, several approaches have been proposed for constructing prefilters.

15   A first approach to prefilter design employs Minimum Mean Square Error (MMSE) techniques. The MMSE prefilter design follows from the design of a pure Decision Feedback Equalizer (DFE). However, the complexity of the MMSE prefilter design is high. Furthermore, the MMSE prefilter may color the noise, making it undesirable as a prefilter for a hybrid MLSE/DFE equalizer since the Viterbi

20   MLSE part is not optimal for signals impacted by colored noise.

Second, allpass filters have been used as prefilters so that the new channel estimate becomes a minimum phase filter. These kinds of prefilters have been designed using a root search algorithm. The root search algorithm consists of an exhaustive sequential search that identifies each root one at a time. For example, in

25   a 7th order channel estimate, seven roots have to be estimated. The first root has to be found using an iterative search method (e.g., Newton Raphson, such as described in Chapter 7 of Söderström and Stoica, System Identification, Prentice Hall 1989). Next, the root search method performs long division of the channel

estimate with the first root. Thus, a 6th order channel remains after the division. A new iterative search, but now based on this 6th order channel, results in a second root. The method continues the process of iterative search and long division until the channel is reduced to a 4th order system. Then, the remaining four roots can be

5   found by closed form equations. After all the seven roots of the channel estimate are calculated, all roots which are outside the unit circle are mirrored inside the unit circle, which results in a set of seven roots inside the unit circle. Based on this set of roots inside the unit circle, a minimum phase channel is constructed. Also, a prefilter is constructed based on this set of roots. However, the indirect nature of

10   the root search algorithm is computationally complex.

        A third approach is described in an article (Gerstacker et al., "An Efficient Method for Prefilter Computation for Reduced-State Equalization", Proceedings of the PIMRC 2000 conference, London, UK, September 18-21, 2000) and related European Patent Application EP 1032170 A1. The third approach consists of

15   calculating a two stage FIR prefilter. The first FIR filter is a matched filter, i.e., the estimated channel conjugated and reversed. The second FIR filter consists of (a FIR approximation of) the inverse of the minimum phase version of the channel filter, which is also conjugated and reversed. The latter FIR filter is calculated from the solution of a set of Yule-Walker equations. This approach is more

20   sensitive to truncation in the FIR approximations due to the two stage filtering, compared to the method described in the present document. Additionally, in section three of the article the authors detail other approaches that have lead to unsatisfactory solutions due to computational complexity. Among these other approaches is a spectral factorization technique which uses the cepstrum of the

25   channel autocorrelation sequence. The authors note that the computational complexity is quite high and requires extensive logarithmic and exponential operations.

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A spectral factorization technique which does not use the cepstrum is described in an article by Jezek, J., and Kucera, V., "Efficient Algorithm for Matrix Spectral Factorization", Automatica, Vol. 21, No. 6, pp. 663-669, 1985. However, this method is designed for real valued filter coefficients and does not

5   allow for complex valued filter coefficients.

Thus, it would be desirable to generate new prefilter designs which are low in complexity, which do not color the noise, and which are implemented in a single stage while at the same time concentrating signal energy in the MLSE taps of the detector.

SUMMARY

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It should be emphasized that the terms "comprises" and "comprising", when used in this specification, are taken to specify the presence of stated features, integers, steps or components; but the use of these terms does not preclude the presence or addition of one or more other features, integers, steps, components or

15   groups thereof.

The present invention describes a spectral factorization method that is a low complexity alternative way of constructing prefilters, requiring a fraction of the computational complexity compared to conventional prefilter designs. Furthermore, the prefilter design of the present invention does not color the signal noise and

20   operates as an allpass filter and can be realized in a single stage. Still further, the required program memory is significantly reduced by the spectral factorization method that does not involve using the cepstrum of the channel autocorrelation sequence or require extensive logarithmic and exponential operations. The spectral factorization method also is less sensitive to quantization noise in signals and

25   channel estimates. Additionally, the invention describes how to construct prefilters to be used for equalization in both the forward and backward direction.

The present invention overcomes the computational complexity and inefficiency of the root search algorithm by directly calculating a minimum phase version, $G(z)$, of a channel estimate, $H(z)$ and calculating the prefilter form the minimum phase. One exemplary method provides for determining filter coefficients in a communication system by estimating a channel over which signals have been transmitted and determining the filter coefficients for a filter from the channel estimator using a spectral factorization method. Another exemplary method begins by initializing coefficients of the minimum phase filter, g, to coefficients of a spectrum of a channel estimate, s, wherein each coefficient numbers from 0 to L. An outer iterative loop starts by calculating a division factor, γ_0, from a complex conjugate of a first coefficient of the minimum phase filter $G(z)$. The outer iterative loop starts by calculating a next L^{th} coefficient, x_L, of the minimum phase filter by multiplying the Lth term of the spectrum with the division factor. A value m (starting at $m = L-1$) of an inner iterative counter is used and the next m^{th} coefficient, x_m, is calculated by an equation as follows:

$$x_m = \left(s_m - \sum_{i=m+1}^{L} x_i g_{i-m}^{'} \right) \gamma_0$$

wherein g^* is the complex conjugate of g. The value m is checked to see if m is equal to zero. If m is not zero, the value m is decreased by one and the process loops back to the start of the inner iterative loop. If m is equal to zero, the coefficients of the minimum phase filter, g, are set equal to the next coefficients of the minimum phase filter, x, for each coefficient from 0 to L. An outer iterative loop criteria is then checked. If the outer iterative loop criteria is met the process ends. If the outer iterative loop criteria is not met the process loops back to the start of the outer iterative loop.

The above features and advantages of the present invention will be more apparent and additional features and advantages of the present invention will be

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appreciated from the following detailed description of the invention made with reference to the drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

The present invention will now be described with reference to the following figures, in which:

Fig. 1 shows a block diagram of the parts of the baseband signal processing with an equalizer of decision feedback type;

Fig. 2 illustrates the prefiltering of the right and left data parts for the GSM system;

Fig. 3 is a flow diagram of the iterations for spectral factorization according to the present invention;

Fig. 4 is a graph of the roots of a channel in a GSM system and the roots of the resulting from the spectral factorization method;

Fig. 5 is a graph of the spectra of a channel in a GSM system and spectra of prefilters from various design methods;

Fig. 6 is a graph of the roots of another channel in a GSM system;

Fig. 7 is a graph of the spectra of a channel in a GSM system for various filters when to few prefilter coefficient are used; and

Fig. 8 is a block diagram of a conventional radio communication system.

DETAILED DESCRIPTION

In the following description, for purposes of explanation and not limitation, specific details are set forth, such as particular circuits, circuit components, techniques, etc. in order to provide a thorough understanding of the present invention. However, it will be apparent to one skilled in the art that the present invention may be practiced in other embodiments that depart from these specific details. In other instances, detailed descriptions of well-known methods, devices,

and circuits are omitted so as not to obscure the description of the present invention.

The exemplary radio communication systems discussed herein are based upon the time division multiple access ("TDMA") protocol, in which

5   communication between the base station and the mobile terminals is performed over a number of time slots. However, those skilled in the art will appreciate that the concepts disclosed herein find use in other protocols, including, but not limited to, frequency division multiple access ("FDMA"), code division multiple access ("CDMA"), or some hybrid of any of the above protocols. Likewise, some of the

10   exemplary embodiments provide illustrative examples relating to the GSM or IS-136 type of systems; however, the techniques described herein are equally applicable to radio communication systems operating in accordance with any specification.

Prior to discussing exemplary embodiments according to the present invention, Fig. 8 will now be described which illustrates a general radio

15   communication system 100 in which the present invention can be implemented. The radio communication system 100 includes a plurality of radio base stations 170a-n connected to a plurality of corresponding antennas 130a-n. The radio base stations 170a-n in conjunction with the antennas 130a-n communicate with a plurality of mobile terminals (e.g. terminals 120a, 120b, and 120m) within a

20   plurality of cells 110a-n. Communication from a base station to a mobile terminal is referred to as the downlink, whereas communication from a mobile terminal to the base station is referred to as the uplink.

The base stations are connected to a mobile telephone switching office ("MSC") 150. Among other tasks, the MSC coordinates the activities of the base

25   station, such as during the handoff of a mobile terminal from one cell to another. The MSC, in turn, can be connected to a public switched telephone network 160, which services various communication devices 180a, 180b, and 180c. Both the mobile terminals 120a, 120b, and 120m, and the base stations 170a-n can

incorporate prefilter design structures and techniques according to the present invention.

A detailed description of prefilter design and prefiltering according to the present invention is given below. The description contains examples using the burst
5   format defined for the GSM standard, but as will be appreciated by those skilled in the art the invention is not limited to this particular standard and can be implemented in communication systems designed in accordance with other standards.

An exemplary block diagram of a receiver chain, and a time dispersive
10   channel over which a signal can be transmitted, is illustrated as Fig. 1. A synchronization and channel estimator unit 11 produces a channel estimate H(z) of the channel 10. The received signal y(n) is prefiltered with the prefilter 12, using filter coefficients established by the transfer function $F_{fwd}(z)$. The output of prefilter 12 results in the signal $y_{fwd}(n)$. The minimum phase (MP) filter $G_{fwd}(z)$, is
15   generated in prefilter design block 16. The minimum phase filter, $G_{fwd}(z)$, is used by the equalizer 14 and has its energy concentrated to the first filter taps.

The prefilter design results in two versions of each filter, one to be used in the forward direction of the burst (with index fwd), and one for the backward direction (with index bwd), as shown in Fig. 2. Therein, data portions defined for
20   frames in the GSM system are illustrated and consist of a ramp block 200 and tail block 202 proceeding the left data burst 204 and training block 206 followed by the right data burst 208, tail 210 and ramp 212 blocks. The filter for the forward direction is used for equalization of the right half of the data burst, from training 206 to ramp 212, and the backward version for equalization of the left half of the
25   burst, from training 206 to ramp 200.

One property of a minimum phase filter G(z) is that all the roots of the filter polynomial are located inside the unit circle. This property will be used in the analysis below. This property of a minimum phase filter will also result in

prefilters which are allpass filters, i.e. in prefilters having a substantially flat frequency response. These prefilters are of an Infinite Impulse Response (IIR) type, with a rational transfer function given as:

$$F(z) = \frac{B(z)}{A(z)} \tag{1}$$

where each root z_i of the numerator $B(z_i) = 0$ is located outside the unit circle. The roots of the denominator are $1/z_i^*$, i.e. the roots of the numerator mirrored in the unit circle. These IIR filters can be approximated by Finite Impulse Response (FIR) filters. In general, IIR filters are avoided in fixed point DSP implementations since rounding errors might result in unstable filters. Furthermore, as will be clarified in the section below regarding prefilter design, this approximation is useful in the design of prefilters based on the spectral factorization method.

An example of a spectral factorization algorithm according to an exemplary embodiment of the present invention is given below. This iterative algorithm is designed to deal with complex valued filter coefficients. Furthermore, the algorithm leads to a simple system of linear equations to be solved for each iteration, thereby facilitating implementation of the prefilter design.

For example, denote the Z-transform of the channel estimate by:

$$H(z) = h_0 z^L + h_1 z^{L-1} + \dots + h_L. \tag{2}$$

This channel estimate is in general not minimum phase. The purpose of the spectral factorization method is to find a minimum phase channel G(z) such that:

$$|H(e^{j\omega})|^2 = |G(e^{j\omega})|^2, \quad 0 \le \omega < 2\pi \tag{3}$$

where:

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$$H(z)\big|_{z = e^{j\omega}} = H(e^{j\omega}). \tag{4}$$

The magnitude response is thus identical for H(z) and G(z) but the phase responses are different. The spectrum of the channel estimate is:

$$S(z) = H(z)H'(z^{-1})$$
$$= s_{-L}z^{L} + s_{-L+1}z^{L+1} + \ldots + s_{0} + \ldots + s_{L-1}z^{-L+1} + s_{L}z^{-L} \tag{5}$$

Since H(z) has complex valued coefficients in general, this spectrum has complex conjugated symmetric coefficients, i.e. $s_{-i} = s_{i}^{*}$.

5   The minimum phase version of the channel estimate is modeled as:

$$G(z) = g_{0}z^{L} + g_{1}z^{L-1} + \ldots + g_{L}. \tag{6}$$

The coefficients of the spectrum S(z) can be expressed through the coefficients of G(z) as:

$$s_{m} = \sum_{i=0}^{L-m} g_{i}^{*}g_{i+m} \quad \text{for } 0 \le m \le L, \tag{7}$$

or, equivalently, in a matrix formulation:

$$s = \begin{bmatrix} s_{0} \\ s_{1} \\ \ldots \\ s_{L} \end{bmatrix} = \begin{bmatrix} g_{0} & g_{1} & \cdots & g_{L} \\ 0 & g_{0} & \cdots & g_{L-1} \\ & 0 & & \cdots \\ 0 & & 0 & g_{0} \end{bmatrix}' \begin{bmatrix} g_{0} \\ g_{1} \\ \ldots \\ g_{L} \end{bmatrix} = T'(g)g. \tag{8}$$

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The criteria which is considered is:

$$V = \left\| s - T'(g^{(k-1)})g^{(k)} \right\| \tag{9}$$

where $g^{(k)}$ denotes the estimated coefficients at iteration k and $g^{(k-1)}$ denotes the previously estimated coefficients. The algorithm for finding a minimum phase channel is based on finding values of the coefficients of $g^{(k)}$ such that the criteria V in equation (9) is minimized. The derivative of V with respect to $g^{(k)}$ is $T*(g^{(k-1)})$.

5 A Newton Raphson search, is thus:

$$g^{(k+1)} = g^{(k)} + T'^{-1}(g^{(k)})(s - T'(g^{(k)})g^{(k)}) = (T'(g^{(k)}))^{-1}s \tag{10}$$

where $g^{(k-1)}$ has been replaced by $g^{(k)}$. Repeat equation (10) until:

$$V = \left\| s - T'(g^{(k-1)})g^{(k)} \right\| < \varepsilon \tag{11}$$

for some value of ε. The stop limit of the criteria, ε, is a design variable the value of which determines the requested accuracy of the minimum phase channel. The

10 vector g is initiated, for k = 0 as:

$$g^{(0)} = s. \tag{12}$$

Those skilled in the art will appreciate that other values, e.g., within the unit circle, can be used to initiate vector g. If the values specified for initialization of the vector g are outside the unit circle, then a maximum phase filter will be generated, from which the minimum phase can be obtained.

15 Experiments have indicated that equation (10) should be iterated approximately 10 times for a GSM channel to provide sufficient accuracy. Those skilled in the art will appreciate that the number of iterations can be greater or less than 10. Spectral factorization methods according to the present invention permit determining complex valued filter coefficient using a relatively few number of

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iterations, e.g., less than about 30. The system of equations in equation (10) can be solved in approximately $\frac{1}{2}L^2$ multiplications. It may be desirable to implement algorithms according to the present invention using a fixed number of iterations instead of permitting a variable number of iterations halted by a limit on the criteria

5   in order to be able to exactly predict the computational requirements for the algorithm.

Solving triangular and Toeplitz systems

The algorithm for finding the spectral factor $G(z)$ is

$$g^{(k+1)} = (T'(g^{(k)}))^{-1}s. \tag{13}$$

Let $x = g^{(k+1)}$, $g = g^{(k)}$ and solve the following system of equations for x:

$$
\begin{bmatrix} s_0 \\ s_1 \\ \cdots \\ s_{L-1} \\ s_L \end{bmatrix}
=
\begin{bmatrix} g_0 & g_1 & \cdots & g_{L-1} & g_L \\ 0 & g_0 & & & g_{L-1} \\ \cdots & & & & \cdots \\ & & & & \\ 0 & & \cdots & 0 & g_0 \end{bmatrix}
\cdot
\begin{bmatrix} x_0 \\ x_1 \\ \cdots \\ x_{L-1} \\ x_L \end{bmatrix}. \tag{14}
$$

10   Starting with the last row of equation (14) yields the solution to x_L as:

$$x_L = s_L/g_0'. \tag{15}$$

The value of x_{L-1} can now be computed as:

$$x_{L-1} = (s_{L-1} - x_L g_1')/g_0'. \tag{16}$$

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The system is solved by iterating upwards in the system of equations (14). Note that the calculations above can be performed using only additions, multiplications and one division. The division is performed by defining:

$$\gamma_0 = 1/g_0^{*} \qquad (17)$$

and inserting γ_0 instead of $1/g_0^{*}$ above. The absolute value of g_0^{*} is not close to zero since it is the first coefficient of a minimum phase filter. The algorithm is illustrated below and in Fig. 3.

Referring to Fig. 3, block 300 initializes $g_i = s_i$ for $m=0,...,L$. Next in block 302, γ_0, x_L and counter m are all initialized. Blocks 304, 306 and 310 form an inner iterative loop that solves equation (14) by sequentially solving for x_m at each iteration. An outer iterative loop is formed by blocks 302, 320, 322 and 324. In decision block 306, when m has counted down to zero, the process proceeds to block 320 and sets $g_i = x_i$, ($g^{(i)} = g^{(i+1)}$, see equations (13) and (14)). The process then continues to block 322 to check if the outer counter k is equal to K. Upon a negative decision in block 322, the process increments counter k in block 324 and feeds the updated values for g_i into block 302. The process then continues until the $k = K$ in decision block 322.

The spectral factorization method, that corresponds to Fig. 3, is presented below in pseudo code to further illustrate the present invention.

```
for m=0 to L
           L-m
    s_m  =  Σ  h_i' h_{i+m}        /* Calculate spectra */
           i=0

    g_m  = s_m                     /* Copy result from s to g */
end
for     k = 0 to K-1
```

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$$\gamma_0 = 1 / g_0^*$$

$$x_L = s_L \gamma_0 \qquad \text{/*The last row of (14)*/}$$

for m = L-1 down to 0   /* Iteration upwards in

(14)*/

$$x_m = \left(s_m - \sum_{i=m+1}^{L} x_i g_{i-m}^* \right) \gamma_0$$

5   end

for m=0 to L   /* Copy result from x to g */

$$g_m = x_m$$

end

end.

Prefilter Design

Forward Prefilter Calculation

A forward prefilter calculation is described wherein the result from the
iterative solution above (coefficients g_i) is denoted as $G_{min}(z)$ which is the minimum
phase version of H(z). This minimum phase filter is used for the DFSE of the right
data burst, i.e. in the forward direction:

$$G_{fwd}(z) = G_{min}(z). \tag{18}$$

The channel estimate is modeled by:

$$H(z) = \prod_{z_i \in Q_{in}} (z - z_i) \prod_{z_i \in Q_{out}} (z - z_i) \tag{19}$$

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where the roots inside and outside the unit circle are denoted by:

$$z_i \in Q_{out} \quad \text{if} \quad |z_i|^2 \geq 1 \quad \text{and} \quad z_i \in Q_{in} \quad \text{if} \quad |z_i|^2 < 1 \quad \text{respectively.} \quad (20)$$

The estimated minimum phase version of H(z) has the following structure:

$$G_{fwd}(z) = \prod_{z_i \in Q_{in}} (z - \tilde{z}_i) \prod_{z_i \in Q_{out}} \left(z - \frac{1}{\tilde{z}_i^{\,*}} \right) \quad (21)$$

where the tilde above in the $\tilde{z}_i$ roots indicates the error in the calculation of G_{min} by the spectral factorization.

5   The prefilter for the forward direction can be expressed as:

$$F_{fwd}(z) = \frac{G_{fwd}(z)}{H(z)} = \prod_{z_i \in Q_{out}} \frac{z - 1/\tilde{z}_i^{\,*}}{z - z_i} \prod_{z_i \in Q_{in}} \frac{z - \tilde{z}_i}{z - z_i} \quad (22)$$

The second factor on the right hand side of equation (22) can be neglected when the estimated G_{min} has roots close to the roots, inside the unit circle, of H(z). Note that $F_{fwd}(z)$ is an allpass filter when the second factor is small, which implies a flat frequency spectrum. The IIR filter in equation (22) is not used directly in causal

10   filtering since the denominator contains roots outside the unit circle. Moreover, this filter is not readily implemented in noncausal filtering either due to the presence of roots inside the unit circle. This IIR prefilter can, instead, be approximated by a FIR filter by using a long division to approximate the denominator of the polynomial. Of course, since this is an approximation it will

15   introduce error. The error in this long division, or approximation of the denominator of the polynomial, typically should decrease as the number of FIR taps

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increases. However, H(z) contains roots both inside and outside the unit circle, which should not be problematic for the long division since G_{fwd} and H(z) contain some common roots inside the unit circle which should cancel each other. However, due to possible estimation errors in the spectral factorization, these roots may not cancel completely resulting in a situation where the error in the approximation of the FIR filter actually increases when the number of FIR taps is increased.

To provide for a better approximation, the present invention provides an alternative technique wherein one first computes:

$$F_f(z) = \frac{H(z)}{G_{fwd}(z)} = f_0 z^M + f_1 z^{M-1} + \cdots + f_M \, , \tag{23}$$

where M is the number of taps in the FIR approximation of the IIR filter. Simulations have indicated that M should be some 2-3 times larger than L. The forward filter F_{fwd} is obtained by reversing the order and conjugation of the coefficients of $F_f(z)$ or, equivalently,

$$F_{fwd}(z) = \alpha_f z^M F_f^*(z^{-1}) \tag{24}$$

where α is a constant which scales the forward filter without changing the filter performance:

$$\alpha_f = \prod_{z_i \in Q_{out}} z_i z_i^* \, . \tag{25}$$

The long division above, in equation (24), is an approximation of an IIR filter with a FIR filter. The error in this approximation will then decrease when the number of coefficients M of the FIR filter increase. It is thus desirable that the

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prefilter F_{fwd} is computed as outlined in equations (23) and (24) instead of equation (22).

Backward Prefilter Calculation

In calculating a backward prefilter, let:

$$\bar{H}(z) = z^{L}H(z^{-1}) \tag{26}$$

5   where the overbar thus denotes a reversing of the coefficients. The overbar also indicates a filter to be used on the reversed signal. This reversed polynomial has roots at:

$$\bar{H}(z) = \prod_{z_i \in Q_{in}} \left(z - \frac{1}{z_i} \right) \prod_{z_i \in Q_{out}} \left(z - \frac{1}{z_i} \right) \tag{27}$$

The backward minimum phase filter is now a filter which is a minimum phase version of $\bar{H}(z)$. This backward filter can be expressed as:

$$\bar{G}_{bwd}(z) = \prod_{z_i \in Q_{in}} (z - \bar{z}_i^{\;\cdot}) \prod_{z_i \in Q_{out}} \left(z - \frac{1}{\bar{z}_i} \right) \tag{28}$$

10   By comparing equations(18),(21) and (28) it follows that the backward minimum phase filter is computed as:

$$\bar{G}_{bwd}(z) = G_{min}^{\;\cdot}(z) \tag{29}$$

i.e. the backward filter is achieved by conjugating the coefficients of the minimum phase version of the estimated channel filter. The prefilter to be used in the backward direction is:

$$\bar{F}_{bwd}(z) = \frac{\bar{G}_{bwd}(z)}{\bar{H}(z)} = \prod_{z_i \in Q_{in}} \frac{z - z_i^*}{z - 1/z_i} \prod_{z_i \in Q_{oul}} \frac{z - 1/\bar{z}_i}{z - 1/z_i}. \tag{30}$$

The second factor on the right hand side of equation (30) can be neglected when the estimated $\bar{G}_{bwd}$ has roots close to the roots, inside the unit circle, of $\bar{H}(z)$. Note that $F_{bwd}(z)$ is an allpass filter, when the second factor is small, which implies a flat spectrum. This prefilter can be approximated by a FIR filter with M taps by using

5 long division. But, as described above, since $\bar{H}(z)$ contains roots both inside and outside the unit circle, this long division is not numerically efficient. To improve on the above long division method, first compute:

$$F_b(z) = \frac{\bar{H}(z)}{\bar{G}_{bwd}(z)} = f_0 z^M + f_1 z^{M-1} + \cdots + f_M. \tag{31}$$

The backward filter F_{bwd} is obtained by reversing the order and conjugation of the coefficients of $F_b(z)$, or, equivalently,

$$\bar{F}_{bwd}(z) = \alpha_b z^M F_b{}'(z^{-1}) \tag{32}$$

10   where:

$$\alpha_b = \prod_{z_i \in Q_{in}} z_i z_i^*. \tag{33}$$

Allpass Properties

The whiteness of prefilters designed according to the spectral factorization methods described herein depends on the number of iterations of equation (10) (and the error in the approximation of the IIR prefilter with an FIR filter) which are

5 preferred. This can be seen in equation (22) and (30) where the accuracy of the spectral factorization is modeled by the closeness of $\hat{z}_i$ to z_i.

In the prefilter design by the root search method, the roots of the channel filter $\hat{z}$ for $i = 1, \ldots, L$, are explicitly calculated. The prefilter is calculated directly from these estimates as:

$$F_{fwd, root} = \prod_{z_i \in Q_{out}} \frac{z - 1/\hat{z}_i^*}{z - \hat{z}_i}. \tag{34}$$

10 The prefilter is then, by construction, an allpass filter, as long as the FIR approximation of equation (34) is not truncated too early. The minimum phase filter is then constructed by a convolution of this prefilter by the original channel filter. To further appreciate various qualities of filter design according to the present invention, consider the following simulation results.

15 Referring to Fig. 4, the roots of a channel filter and corresponding roots mirrored in the unit circle for a Hilly Terrain channel, as defined by GSM specification 5.05, are shown. Three sets of symbols illustrate these roots. The first set of symbols are the roots of channel filter (+), such as 420. The second set of symbols are the roots mirrored inside the unit circle (x), such as 410. The third

20 set of symbols are the roots of the minimum phase filter found by the spectral factorization, as described above, are displayed as circles (o), such as 430. The roots found by the spectral factorization method were identified by iterating equation (10) ten times. Fig. 4 illustrates a minimum phase filter and its relationship to a channel filter that is not minimum phase. The spectral

25 factorization method yields, as is shown by a comparison of the roots, essentially

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the same minimum phase filter. The spectral factorization method of the present invention achieves this result using significantly less processing power than conventional methods. All roots of the channel filter are not shown in Fig. 4, because one root lies significantly outside the unit circle. Table 1 lists the values as plotted in Fig. 4, and all the roots for the channel filter.

Roots of the channel filter	Roots mirrored in the unit circle	Roots of the minimum phase filter
0.5489 + 0.5581i	0.8958 + 0.9107i	0.5489 + 0.5581i
0.4720 - 0.6543i	0.7384 - 1.0095i	0.4720 - 0.6453i
-0.7338 + 0.6072i	-0.8089 + 0.6694i	-0.7338 + 0.6072i
-0.8871 - 0.5104i	-0.8469 - 0.4873i	-0.8469 - 0.4873i
-3.2978 + 0.1205i	-0.3028 + 0.0111i	-0.3028 + 0.0111i

Table 1.

The graph of Fig. 5 shows the spectrum of the channel filter 502, which corresponds to the channel filter of Fig. 4. The spectra of the prefilters from the root search algorithm 504 and from the spectral factorization 506 are also shown. It can be seen that the spectrum of the root search 504 is flatter than the spectrum of the spectral factorization prefilter 506. The minimum phase filter estimate becomes more accurate as the number of iterations of the spectral factorization is increased. An increase in the number of iterations also results in a flatter ("whiter") spectrum of the prefilter, i.e. a more ideal allpass response. However, an increase in the number of iterations also increases processing time. Therefore, the number of iterations performed should provide an acceptable result, while minimizing processing time, especially for real time systems.

Referring to Fig. 6, the roots of a channel filter and corresponding roots mirrored in the unit circle for a different channel simulation are shown. Three sets

of symbols illustrate these roots. The first set of symbols are the roots of channel filter (+), such as 610. The second set of symbols are the roots mirrored inside the unit circle (x), such as 620. The third set of symbols are the roots of the minimum phase filter found by the spectral factorization, as described above, are displayed as

5 circles (○), such as 630. The roots found by the spectral factorization method were identified by iterating equation (10) ten times. Fig. 6 illustrates a minimum phase filter and its relationship to a channel filter that is not minimum phase for another channel simulation. The spectral factorization method yields, as is shown by a comparison of the roots, essentially the same minimum phase filter. All roots of

10 the channel filter are not shown in Fig. 6, because two roots lie significantly outside the unit circle. Table 2 lists the values as plotted in Fig. 6, and all the roots for the channel filter.

Roots of the channel filter	Roots mirrored in the unit circle	Roots of the minimum phase filter
0.9970 + 0.2052i	0.9622 + 0.1981i	0.9622 + 0.1981i
-0.3429 + 1.0461i	-0.2830 + 0.8631i	-0.2830 + 0.8631i
0.1142 - 1.1586i	0.0843 - 0.8548i	0.0843 - 0.8548i
-1.2868 - 0.1622i	-0.7650 - 0.0964i	-0.7650 - 0.0964i
-2.6979 - 0.6838i	-0.3483 - 0.0883i	-0.3483 - 0.0883i

Table 2.

Referring to Fig. 7, another trade-off in the construction of minimum phase prefilters is illustrated. This problem arises when the roots are located close to the unit circle. The spectra of the channel filter 702 is plotted with the spectra of both the root search algorithm 704 and the spectral factorization 706 prefilter designs.

25 The spectrum of the channel filter 702 corresponds to the channel filter of Fig. 6. The spectra of both the root search algorithm 704 and the spectral factorization 706 prefilter designs have large dips, 710 and 708, respectively. The dips 708 and 710 become smaller as the number of taps in the prefilter increases. In this example, it

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can be seen that the long division in equation (23) (and equation (31)) is truncated too early. The "whiteness" of the prefilter can thus be improved by increasing the number of taps of the prefilter. The prefilters of both Fig. 5 and Fig. 7 have the same number of taps. However, due to the different channel filters, the number of taps in the latter case was not sufficient. Although the root search algorithm is much more computationally complex and exhaustive, as shown in Fig. 7, the resulting prefilter is not better than that achieved by the spectral factorization method of the current invention.

The invention provides a design method for calculating minimum phase prefilters. The method is based on using a spectral factorization approach. The prefilter is constructed from a minimum phase version of an estimated channel impulse response. A system of linear equations is solved for each of the iterations of the spectral factorization. The invention is, unlike most other spectral factorization methods, designed to deal with complex valued filter coefficients. From an implementation point of view, the present invention is appealing because it requires only multiplications, additions and one division for each of the iterations. The proposed method of calculating the prefilter becomes very important as the number of prefilter coefficients are increased. Additionally, the invention describes how to construct prefilters to be used for equalization in both the forward and backward direction. The reduced complexity of the current prefilter design allows for the prefilter to be implemented on processors with reduced computing power.

The minimum phase filter improves the performance of an equalizer using decision feedback. The invention makes the implementation of the minimum phase filter design possible in fixed point Digital Signal Processors (DSPs).

The foregoing examples have been described in the context of prefilters used in hybrid MLSE/DFE systems. However, those skilled in the art will appreciate that the present invention can be implemented in systems using any type of equalizers, e.g., pure MLSE equalizers.

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Therefore, the above-described embodiments should be regarded as illustrative rather than restrictive, and it should be appreciated that variations may be made in those embodiments by workers skilled in the art without departing from the scope of the present invention as defined by the following claims.

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CLAIMS

What is claimed is:

1.   A method for determining filter coefficients in a communication system comprising the steps of:

        estimating a channel over which signals have been transmitted; and

        determining said filter coefficients for a filter from said channel estimates

5   using a spectral factorization algorithm.

2.   The method of claim 1, wherein said spectral factorization algorithm generates at least one complex valued filter coefficient.

3.   The method of claim 2, wherein said step of determining said filter

10   coefficients using spectral factorization algorithm comprises the step of:

        interatively solving a system of linear equations.

4.   The method of claim 3, wherein said step of determining further comprises the step of:

        performing less than 30 iterations to determine said filter coefficients.

15   5.   The method of claim 3, wherein solving the system of linear equations comprises the steps of:

        (a)   determining spectrum coefficients of a spectrum of said channel estimate;

        (b)   initializing current filter coefficients;

20   (c)   creating a complex conjugate matrix containing complex conjugates of the current filter coefficients;

(d)   multiplying the inverse of said complex conjugate matrix by a vector containing said spectrum coefficients to obtain next filter coefficients;

(e)   setting said current filter coefficients as equal to said next filter coefficients; and

5   (f)   checking a loop criteria, wherein if said loop criteria is met ending, else looping to step (c).

6.   The method of claim 5, wherein said step of initializing comprises the step of:

setting said current filter coefficients equal to said spectrum coefficients.

10   7.   The method of claim 5, wherein said loop criteria is met by looping a fixed number of times.

8.   The method of claim 5, wherein said loop criteria is met by:

$$V = \left\| s - T'(g^{(k-1)})g^{(k)} \right\| < \varepsilon$$

9.   The method of claim 1, wherein said filter coefficients are a minimum phase version of said channel estimate.

15   10.   The method of claim 1, wherein said filter coefficients are a maximum phase version of said channel estimate.

11.   The method of claim 1, further comprising the steps of:

determining at least one of forward and backward prefilters using said filter coefficients.

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12. The method of claim 11, further comprising the steps of:

approximating at least one of forward and backward prefilters as Finite Impulse Response (FIR) filters.

13. The method of claim 12, wherein the step of approximating at least one of

5 the forward and backward prefilters further comprises the steps of:

determining intermediate coefficients for at least one prefilter;

reversing the order of said intermediate coefficients for at least one prefilter;

conjugating said intermediate coefficients for at least one prefilter; and

scaling said reversed and conjugated intermediate coefficients to determine

10 at least one prefilter.

14. The method of claim 1:, wherein determining the forward prefilter further comprises the steps of:

calculating the forward prefilter according to:

$$F_f(z) = \frac{H(z)}{G_{fwd}(z)} = f_0 z^M + f_1 z^{M-1} + \cdots + f_M$$

and

$$F_{fwd}(z) = \alpha_f z^M F_f^{\,*}(z^{-1}).$$

15 15. The method of claim 11, wherein determining the backward prefilter further comprises the steps of:

calculating the backward prefilter according to:

$$F_b(z) = \frac{\overline{H}(z)}{\overline{G}_{bwd}(z)} = f_0 z^M + f_1 z^{M-1} + \cdots + f_M$$

and

$$\bar{F}_{bwd}(z) = \alpha_b z^M F_b'(z^{-1}).$$

16. The method of claim 1, further comprising the steps of:

receiving a channel input;

determining a channel estimate using said channel input

supplying said channel estimate to said spectral factorization algorithm; and

supplying said filter coefficients to an equalizer.

17. The method of claim 1, further comprising:

determining both forward and backward prefilters with said filter

coefficients.

18. The method of claim 1, wherein said spectral factorization algorithm does

not use the cepstrum of the channel autocorrelation sequence.

19. The method of claim 1, wherein said filter coefficients are determined in a

single stage FIR approximation.

20. A transceiver in a communication system comprising:

a filter;

a channel estimator that estimates a channel over which signals have been

transmitted; and

a filter design unit that performs a spectral factorization algorithm that

determines filter coefficients from said channel estimates, wherein said filter

coefficients are used to calculate said filter.

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21. The transceiver of claim 20, wherein said transceiver is base station transceiver.

22. The transceiver of claim 20, wherein said transceiver is mobile terminal transceiver.

5   23. The transceiver of claim 20, wherein said spectral factorization algorithm generates at least one complex valued filter coefficient.

24. The transceiver of claim 20, wherein said filter coefficients are a minimum phase version of said channel estimate.

25. The transceiver of claim 20, wherein said filter coefficients are a maximum
10   phase version of said channel estimate.

26. The transceiver of claim 20, wherein a prefilter has both forward and backward prefilters that are designed using said filter coefficients.

27. The transceiver of claim 20, further comprising:
     an equalizer for compensating said radio signals for effects of said channel.

15   28. The transceiver of claim 27, wherein said equalizer is a Decision Feedback Sequence Estimator (DFSE) equalizer.

29. The transceiver of claim 28, wherein said filter is a prefilter that focuses energy of said radio signals in taps associated with an MLSE portion of said DFSE equalizer.

30. The method of claim 20, wherein said spectral factorization algorithm does not use the cepstrum of the channel autocorrelation sequence.

31. The method of claim 20, wherein said filter coefficients are determined in a single stage FIR approximation.

5   32. A method of calculating a minimum phase version, G(z), of a channel estimate, H(z), for implementing a prefilter using spectral factorization, comprising the steps of:

(a) initializing coefficients of the minimum phase filter, g, to coefficients of a spectrum of a channel estimate, s, wherein each said coefficients number from 0

10   to L;

(b) calculating a division factor, γ_0, from a complex conjugate of a first coefficient of said minimum phase filter;

(c) calculating a next Lth coefficient, x_L, of the minimum phase filter by multiplying the Lth of said spectrum with said division factor;

15   (d) establishing a value m of an inner iterative counter as one less than L;

(e) calculating the next mth coefficient, x_m, by an equation as follows:

$$x_m = \left(s_m - \sum_{i=m+1}^{L} x_i g_{i-m}^{*} \right) \gamma_0$$
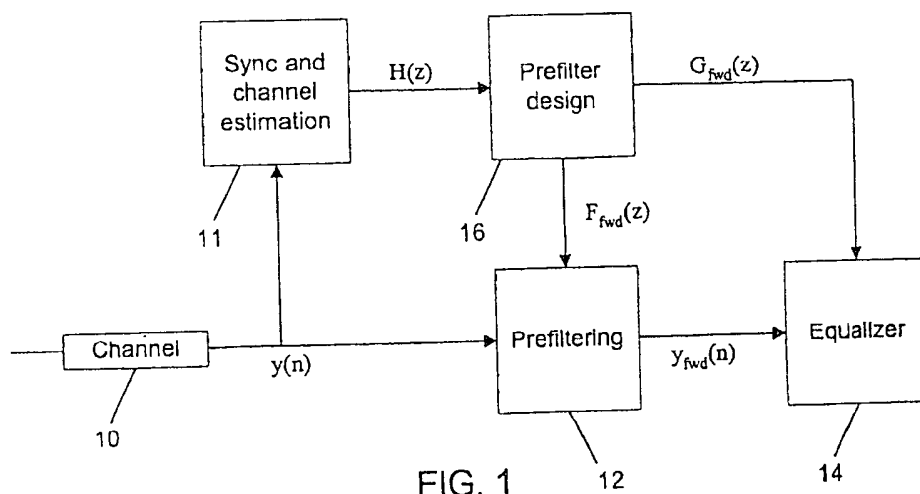
wherein g^* is the complex conjugate of g;

20   (f) checking the value m, wherein if the value m is equal to zero continue to step (g), else decrement the value m by one and loop to step (e);

(g) equating the coefficients of the minimum phase filter, g, to the next coefficients of the minimum phase filter, x, for each coefficient from 0 to L; and

(h) checking an outer iterative loop criteria, wherein if said outer iterative loop criteria is met ending, else loop to step (b).

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FIG. 1
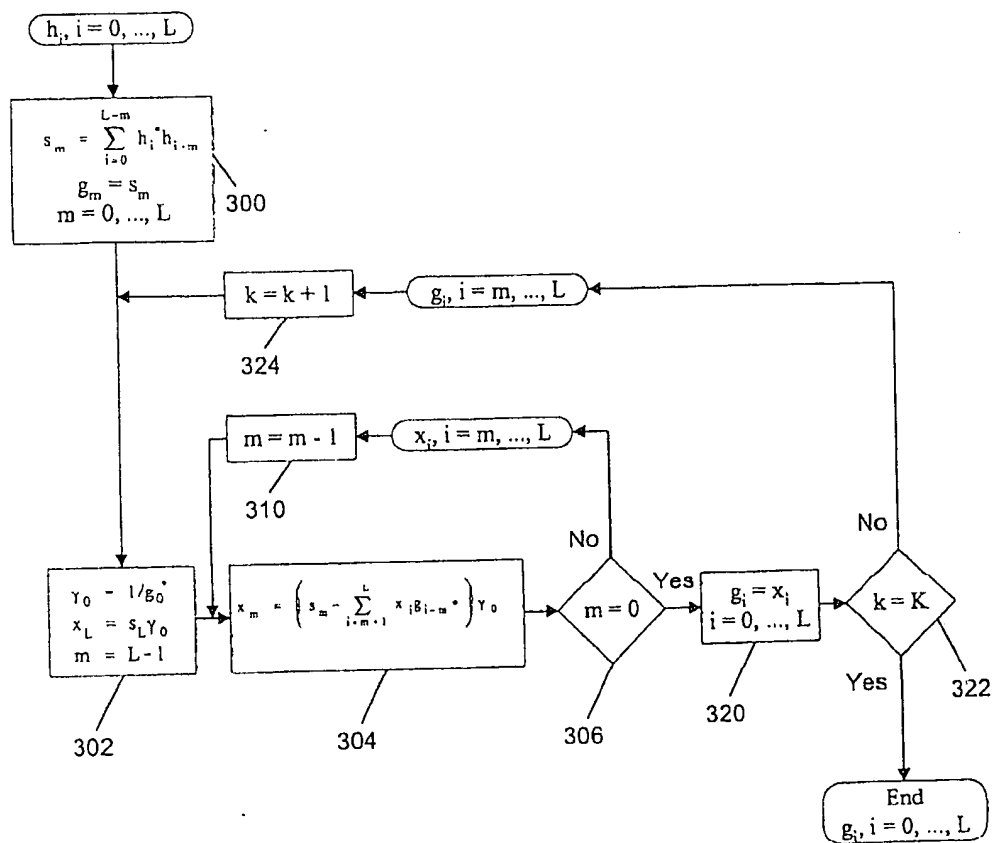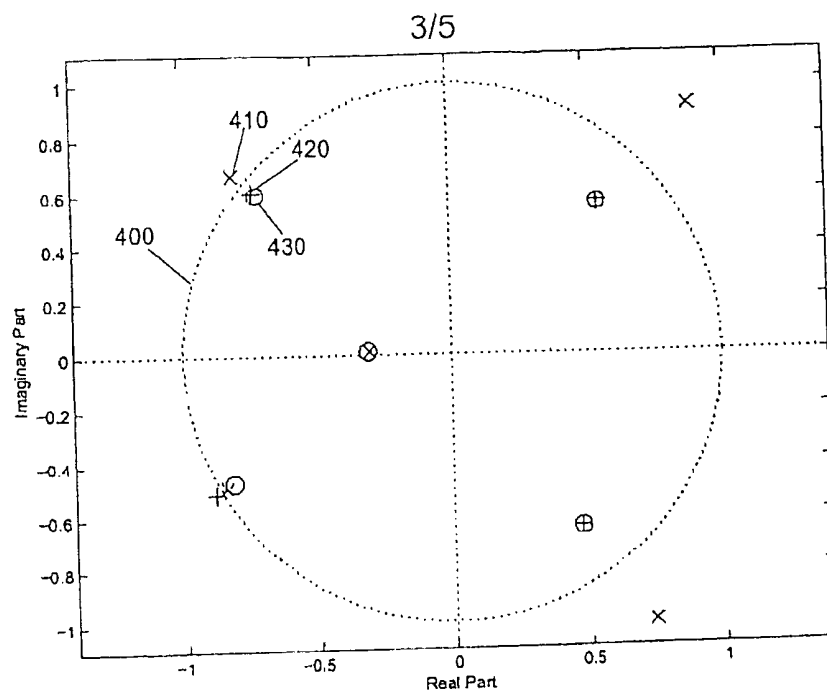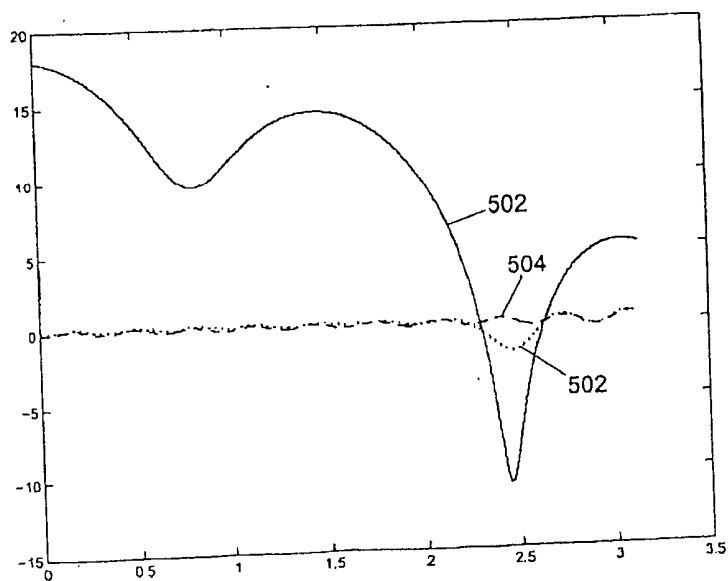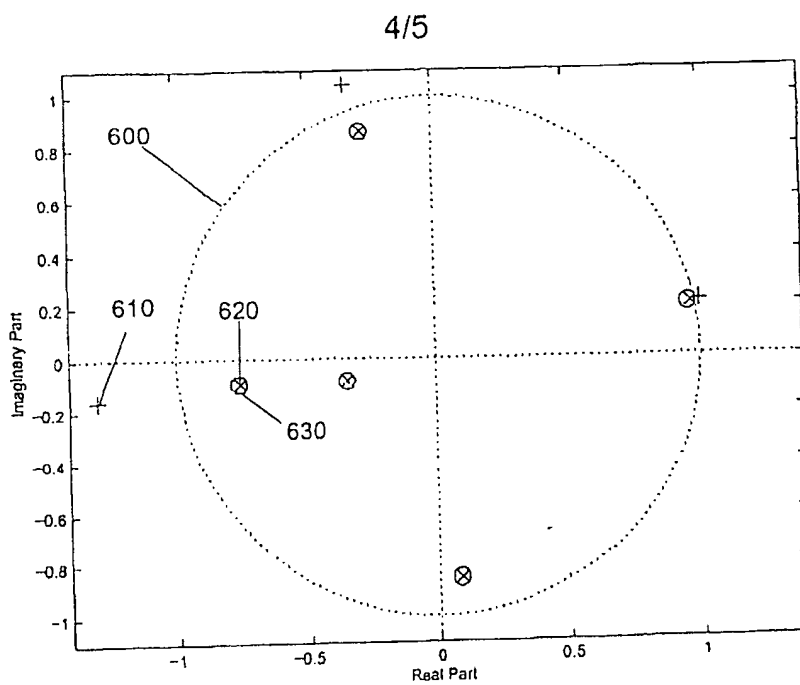

FIG. 2

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FIG. 3

FIG. 4

FIG. 5

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FIG. 6

FIG. 7

FIG. 8